# APPARENT MASS COEFFICIENT IN HORIZONTAL HYDRODYNAMIC IMPACT OF A FLOATING SPHERE

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**ABSTRACT:** The practical three-dimensional problem of horizontal hydrodynamic impact of a floating body was examined for the first time by E. L. Blokh [1] who obtained a solution for the case of a sphere half-submerged in an incompressible fluid. V. I. Mosakovskii and V. L. Rvachev [2] obtained a solution of the same problem in closed form.

The results of [1, 2] are extended below to the case of an arbitrary depth of submergence. As in [1, 2], it is considered that there is no separation of the fluid from the wetted surface of the sphere.

§1. Let a sphere of unit radius $x^2 + y^2 + (z - h)^2 = 1$ float in an ideal fluid filling the half-space $z \geq 0$. As a result of a suddenly applied impulsive force, the sphere, which at first is not moving is set in translational motion along the x-axis with a speed $U_0$. Then [3], in the absence of mass impulsive forces, the motion of the fluid is potential after the impact, and the velocity potential $\varphi^*$ is a harmonic function connected with the impulsive pressure $p_t$ by the relationship $p_t = -\rho\varphi^*$, where $\rho$ is the density of the fluid.

On the free surface of the fluid

$$\varphi^* = 0. \tag{1.1}$$

On the wetted surface of the sphere, on the strength of the assumption of impact without separation

$$\partial\varphi^* / \partial n = v_n. \tag{1.2}$$

Here $v_n$ is the projection on the normal to the surface of the velocity of points on the surface.

At infinity, the fluid is not in motion, and

$$\operatorname{grad} \varphi^* = 0. \tag{1.3}$$

The potential flow of the fluid is defined uniquely by conditions (1.1)–(1.3).

§2. Let $|h| < 1$. We introduce the toroidal coordinates

$$x = \frac{c \operatorname{sh} \alpha \cos \gamma}{\operatorname{ch} \alpha - \cos \beta}, \qquad y = \frac{c \operatorname{sh} \alpha \sin \gamma}{\operatorname{ch} \alpha - \cos \beta}, \qquad z = \frac{c \sin \beta}{\operatorname{ch} \alpha - \cos \beta}.$$

If $\beta = \beta_0$ is the equation of the wetted part of the sphere, then $h = \cos \beta_0$ and $c = \sin \beta_0$. The free surface of the fluid has the equation $\beta = 0$. The boundary conditions (1.1) and (1.2) take the form

$$\varphi^{\cdot} = 0, \quad \beta = 0, \quad \frac{\partial\varphi}{\partial\beta} = -\frac{c^2 U_0 \operatorname{sh} \alpha \cos \gamma}{(\operatorname{ch} \alpha - h)^2}, \quad \beta = \beta_0.$$

We shall seek the solution in the form of an expansion into a generalized Meller-Fok integral [4] with respect to the associated Legendre functions

$$\varphi(\alpha, \beta, \gamma) = c^2 U_0 \cos \gamma \sqrt{\operatorname{ch} \alpha - \cos \beta} \int_0^\infty A(\tau) \operatorname{sh} \beta\tau P^1_{-1/2+i\tau}(\operatorname{ch} \alpha)\, d\tau. \tag{2.2}$$

In this case, the first condition of (2.1) is satisfied, and the second condition is satisfied if

$$A(\tau) = \frac{F(\tau)\sqrt{\operatorname{ch} \alpha - h}}{\tau \operatorname{ch} \beta_0\tau (\operatorname{ch} \alpha - m_0)}$$

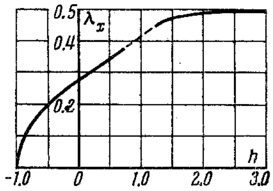
$$\left(F(\tau) = \frac{\pi\tau \operatorname{th} \pi\tau}{\operatorname{ch} \pi\tau} P^1_{-1/2+i\tau}(h), m_0 = h - \frac{c}{2\tau} \operatorname{th} \beta_0\tau\right).$$

In finding $A(\tau)$, the left side of (2.2) was expanded into an integral with respect to the associated functions, which is achieved by differentiating the following relationship with respect to $\alpha$:

$$\frac{1}{\operatorname{ch} \alpha - h} = \int_0^\infty F(\tau) P_{-1/2+i\tau}(\operatorname{ch} \alpha)\, d\tau. \tag{2.3}$$

In particular, on the wetted surface

$$\varphi(\alpha, \beta_0, \tau) = -2cU_0 \left[\frac{\operatorname{sh} \alpha}{\operatorname{ch} \alpha - h} + (\operatorname{ch} \alpha - h)^2 \int_0^\infty \frac{F(\tau)}{\operatorname{ch} \alpha - m_0} P^1_{-1/2+i\tau}(\operatorname{ch} \alpha)\, d\tau\right] \cos \gamma. \tag{2.4}$$




§3. Let $h > 1$. We introduce the bispherical coordinates

$$x = \frac{c \sin \alpha \cos \gamma}{\operatorname{ch} \beta - \cos \alpha}, \quad y = \frac{c \sin \alpha \sin \gamma}{\operatorname{ch} \beta - \cos \alpha}, \quad z = \frac{c \operatorname{sh} \beta}{\operatorname{ch} \beta - \cos \alpha}.$$

Let $\beta = \beta_1$ be the equation of the sphere $c = \operatorname{sh} \beta$, and $h = \operatorname{ch} \beta_1$. The equation of the free surface is $\beta = 0$. The boundary conditions (1.1) and (1.2) are written in the form

$$\varphi = 0, \quad \beta = 0, \quad \beta = \beta_1, \quad \frac{\partial\varphi}{\partial\beta} = -\frac{c^2 U_0 \sin \alpha \cos \gamma}{(h - \cos \alpha)^2}. \tag{3.1}$$

Here $\varphi(\alpha, \beta, \gamma) = \varphi^*(x, y, z)$. We shall seek the solution in the form of a series [5]

$$\varphi(\alpha, \beta, \gamma) = c^2 U_0 \cos \gamma \sqrt{\operatorname{ch} \beta - \cos \alpha} \sum_{n=1}^\infty B_n \operatorname{sh}\left(n + \frac{1}{2}\right) \beta_1 P_n^1(\cos \alpha). \tag{3.2}$$

The first condition of (3.1) is satisfied. In order to satisfy the second condition, we differentiate the known expansion with respect to $\alpha$

$$\frac{1}{h - \cos \alpha} = \sum_{n=0}^\infty (2n + 1) Q_n(h) P_n(\cos \alpha), \tag{3.3}$$

and comparing the result with $\partial\varphi/\partial\beta_1$ obtained from (3.2), we find that

$$B_n = \frac{2Q_n(h)\sqrt{h - \cos \alpha}}{\operatorname{ch}(n + 1/2)\beta_1 (m - \cos \alpha)}$$

$$\left(m_1 = h + \frac{c}{2n+1} \operatorname{th}\left(n + \frac{1}{2}\right)\beta_1\right).$$

In particular, the expression for the potential on the surface of the sphere is of the form

$$\varphi(\alpha, \beta, \gamma) = -2cU_0 \cos \gamma \left[\frac{\sin \alpha}{h - \cos \alpha} + (h - \cos \alpha)^2 \sum_{n=1}^\infty \frac{(2n+1) Q_n(h)}{m_1 - \cos \alpha} P_n^1(\cos \alpha)\right]. \tag{3.4}$$

§4. The apparent mass coefficient of the sphere acting along the x-axis

$$\lambda_x = -\frac{P_t}{\rho V U_0}\left(P_t = -\iint_{(s)} p_t \cos(n, x)\, ds\right).$$

Here $P_t$ is the resultant of the impulsive pressure forces directed opposite to the motion of the sphere; V is the volume of the displaced fluid.

In the case of a partially submerged sphere, we have

$$\lambda_x(h) = 2 - 12\frac{(1-h)^2}{c(2-h)}\int_0^\infty \tau^2 \frac{\operatorname{th}\pi\tau}{\operatorname{th}\beta_0\tau}\, d\tau \int_0^\infty \frac{\cos\tau t}{(\operatorname{ch} t - h)^{1/2}}\, dt \times$$
$$\times \int_0^\infty \left[\frac{c^2}{(\operatorname{ch} s - h)^{3/2}} + \frac{m_0^2 - 1}{(\operatorname{ch} s - m_0)^{3/2}}\right] \cos\tau s\, ds. \qquad (4.1)$$

In these calculations, we make use of known integral representations of associated Legendre functions [5].

If h > 1, then

$$\lambda_x(h) = 2 +$$
$$+ 3c^3 \sum_{n=1}^{\infty} \frac{(2n+1)^2}{\operatorname{th}(n+{}^1/_2)\beta_1} Q_n(h)\left[c^2 Q_n'(h) - (m_1^2 - 1)\, Q_n'(m_1)\right]. \quad (4.2)$$

Taking the asymptotic behavior of the functions $Q_n$ and $Q_n'$ into consideration, we can obtain from (4.2) $\lambda_x(\infty) = 1/2$, which corresponds to a sphere in an unbounded fluid [5].

The results of the calculations performed with formulas (4.1) and (4.2) with a relative error $\delta < 2\%$ are presented in the figure.

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